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A two-dimensional mathematical model of annular-dispersed and dispersed flows—I

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Abstract—A two-dimensional mathematical model of two-phase flow is presented. The analytical formulation of the model involves the mass, momentum and energy conservation equations for vapour and droplet flows, liquid film and for the wall of the channel, and also a number of subsidiary relations incorporated to close the set of equations. The assumptions invoked are analysed.

1. INTRODUCTION

IN RECENT times a considerable amount of attention has been devoted to the development of computational methods to determine different two-phase parameters. The need for their development to investigate two-phase flows is accounted for by the fact that it is not always possible to obtain experimentally information required for the design of heat exchangers.

Two-phase flows are characterized by a stepwise variation of physical properties, such as viscosity, density, thermal conductivity, etc., in space and time and, in a general case, by the unsteady and non-uniform character of interphase exchange processes. These features make it difficult to develop mathematical models of two-phase systems. The analysis of available publications shows that the first models of two-phase fluid flows in pipes have been developed not to investigate directly the characteristics of these flows, but to describe analytically the conditions of burnout heat transfer. The basic equation in this case was that for the film flow rate, whereas the flow characteristics were described very schematically and empirically.

The basic parameters of two-phase systems, such as temperature, concentration and phase velocity fields, are determined with the aid of one- and multi-dimensional models based on mass, momentum and energy conservation laws in the form of differential equations. The controversial point here turns out to be the construction of the conservation equations themselves. Two methods of deriving such partial differential equations are known. The first of these, the statistical approach, is characterized by the use of conservation laws in integral form with subsequent space-, time- and ensemble-averaging and transition from integral expressions to differential ones [10–20]. The difficulty of this approach is associated with the determination of the scales of averaging. Here, just as in the theory of turbulence, a non-closed set of equations results whose closure requires resorting to a number of hypotheses. The second method consists in averaging the fluid properties over an isolated space and assuming the interacting phases to be interpenetrating continua [21–27]. No physico-mechanical model is generally applied in this case which would describe the newly obtained hypothetic flow.

The difficulties characteristic of the above-mentioned methods reside in the specification of the conditions for mechanical and thermal interaction of phases and the conditions at the boundaries. As noted in ref. [28], a not very lucid formulation of the boundary conditions is attributable to the fact that the wall region of a two-phase flow is very provisionally modelled by a flow with 'spread' characteristics because of the compliance of the linear scales of motion with the dispersed phase dimensions and because the conditions of zero velocity at the walls for a dispersed phase are not met.

Despite the quantity of works dealing with the derivation of mass, momentum and energy conservation equations for two-phase flows, the number of specific models of two-phase flows are very limited. These are first of all the one-dimensional models which determine the cross-section-averaged basic flow characteristics (phase velocities, vapour temperature, mean film thickness, etc.) and the parameters in the burnout heat transfer cross-section [29, 30]. A characteristic feature of these models is the application of non-universal empirical relations. A comprehensive survey of these works is made in ref. [31] which also considers various techniques for deriving averaged two-phase flow equations needed for the formulation of a two-dimensional problem.

A few two-dimensional models have been developed for the solution of separate problems with momentum, mass and energy conservation laws with a different degree of completeness.

In works [6, 32] one of the most important processes characteristic of annular-dispersed and dispersed flows—the process of dispersed phase motion in a turbulent vapour-droplet core—is modelled in a two-dimensional statement. In ref. [32] the assumptions are substantiated and the limits are specified for the applicability of the diffusion scheme of particle deposition in a turbulent gas flow. It is assumed that

NOMENCLATURE

a	thermal diffusivity
c, C	concentration
c_p	heat capacity
d	diameter
D	diffusion coefficient
g	acceleration due to gravity
I	dimensionless specific mass flux
j	integral mass flux
L	length of the channel
m	liquid flow rate
M	dimensionless liquid flow rate
p	pressure
q	specific heat flux
r, z	transversal and longitudinal coordinate, respectively
r_v	heat of vaporization
S	slip
t	temperature
u, U	vapour velocity
v, V	droplet velocity
x	steam quality
z_0	coordinate of reference point.

Greek symbols

α	heat transfer coefficient
β	hydraulic resistance coefficient
δ	film thickness
Δ	dimensionless film thickness
ε_v	turbulent diffusion coefficient
θ	dimensionless temperature
λ	thermal conductivity
ν	kinematic viscosity
ξ, η	dimensionless transversal and longitudinal coordinate, respectively
ρ	density
σ	surface tension
τ	shear stress
φ	void fraction
Ψ	coefficient taking account of liquid fraction in the core
Ψ_T	dimensionless transport coefficient
Ω	angular velocity
$\kappa, \kappa_1, \kappa_2, N_S, m, P, Q, a, k(x), N, n_1, \Delta_1, \Phi(d_d), f(\xi), F(\xi), k, f_\eta$	coefficients and functions within the text.

Subscripts

b	probable quantities
c	core
cell	cell
d	droplet
dep	deposition
en	entrainment
ev	evaporation
f	film
F	superficial quantities
l	liquid
lin	linear quantities
max	maximum parameters
$0, L$	refer to quantities at the reference point and at the end of the channel
r, ξ	refer to transversal components
rel	relative quantities
s	refers to quantities in saturation conditions
sur	refers to surroundings
t, v	turbulent viscous quantities
v	vapour
V	volumetric quantities
w	wall
z, η	refer to longitudinal component
1, 2, 3	refer to wall, film and core
,	refers to fluctuational quantities.

Superscripts

$\bar{}$	averaged quantities
$'$	refers to liquid
$''$	refers to vapour.

the concentration of admixtures and the sizes of particles are small, the kinematic properties and the transport coefficients of particles and of the medium are identical, and that the admixtures do not alter the turbulent properties of the carrying flow. With these assumptions, the number density distribution of non-inertia particles is described by a two-dimensional diffusion equation with appropriate boundary conditions. The problem formulated in ref. [32] is analytically solved in ref. [6]. Additional assumptions were made in order to introduce the cross-section-constant transport coefficient and the longitudinal velocity component of the carrying medium independent of the transversal coordinate. The transversal flux of the mass of droplets obtained from the solution of the above problem was used to construct an analytical relation to determine the critical heat fluxes.

In ref. [33] a model of a non-equilibrium dispersed flow is presented. The model is based on two-dimensional mass, momentum and energy conservation equations for vapour flow stated under the following basic assumptions: the processes are steady; direct heat transfer from a wall to liquid is neglected; the liquid phase is uniformly distributed over the tube cross-section; the temperature of droplets is equal to the saturation temperature; interphase interactions are ignored.

Below, a two-dimensional model of annular-dispersed and dispersed flows is presented in the form of a specified and closed set of partial differential equations. The construction of problems to determine the temperature, velocity and phase concentration fields is considered consistently, the hypotheses introduced are substantiated, and the results obtained are discussed.

2. BASIC ASSUMPTIONS

Consider a vertical, electrically heated tube with downstream, alternating, annular-dispersed (conventional heat removal region) and dispersed (post-burnout region) modes of flow.

To analyse in detail the processes occurring in the system, the latter is divided into the following zones: a heat-emitting wall, a two-phase film, and a two-phase core.

The two-dimensional partial differential mass, momentum and energy conservation equations for each phase in the zones (for the wall, only the energy equation) in unsteady-state formulation are given in ref. [34]. The system was specified for the given geometry by the procedure suggested in ref. [15]. However, the resulting complete mathematical model did not form a closed system, and the problem of its numerical solution remained unsolved.

In order to close and numerically solve the system, it was simplified under the following assumptions:

(1) The problem is of a steady-state nature.
(2) The temperature field of the heat-emitting wall is the function of the longitudinal coordinate alone.
(3) The fraction of heat due to film superheating is not taken into account in the general enthalpy balance.
(4) The liquid film is a single-phase (non-boiling) film and the core–film interface is not disturbed (no rippling). These assumptions are quite suitable for the regions close to the burn-out cross-section whose boundaries can be defined in terms of Re_f.
(5) In the diffusion equation the entrainment flow is considered as mass sources distributed in a given manner. The hypothesis is justifiable when remembering that the droplets entrained from the film surface scatter the initial momentum, while interacting with the flow at some distance from the wall and, starting from a certain coordinate (the place of mass source action), move only due to the forces acting upon a droplet in a turbulent gas flow.
(6) In the vapour-droplet core there are differently sized droplets and, to take into account the influence of the droplet size on diffusive and convective motion of droplets in a flow, the entire assembly of droplets is treated as an aggregate with equally sized droplets in each group; the mass flow or the number density of droplets is considered as additive quantities of the corresponding characteristics of separate groups.
(7) When formulating the problem on the determination of velocity fields in the core and the film, the vapour-droplet flow core is represented as a homogeneous Newtonian fluid flow with known properties.

3. MATHEMATICAL MODEL

3.1. The one-dimensional energy equation for a tube wall with internal energy release, familiar axial heat flux distribution and efflux of heat in the initial and final cross-sections is

$$\frac{\partial}{\partial z}\left[(r_2^2-r_0^2)\lambda_{z1}\frac{\partial t_1}{\partial z}\right]+q_{V1}(r_2^2-r_0^2)-2r_0q_{F1}=0 \quad (1)$$

with the boundary conditions

$$-\lambda_{z1}\frac{\partial t_1}{\partial z}=-\alpha_{01}(t_1-t_{0,\mathrm{sur}}) \quad \text{at} \quad z=z_0 \quad (2)$$

$$-\lambda_{z1}\frac{\partial t_1}{\partial z}=-\alpha_{L1}(t_1-t_{L,\mathrm{sur}}) \quad \text{at} \quad z=L. \quad (3)$$

The two-dimensional energy equation for a vapour-droplet flow core, with heat sinks due to the evaporation of liquid droplets in a superheated vapour taken into account, is

$$(c_p\rho)_3\left(u_{z3}\frac{\partial t_3}{\partial z}+u_{r3}\frac{\partial t_3}{\partial r}\right)=\frac{1}{r}\frac{\partial}{\partial r}r\lambda_{r3}\frac{\partial t_3}{\partial r}-q_{V3} \quad (4)$$

with the boundary conditions

$$t_3=t_s \quad \text{at} \quad z=z_0, \quad 0\leqslant r\leqslant r_1 \quad (5)$$

$$\frac{\partial t_3}{\partial r}=0 \quad \text{at} \quad r=0, \quad z_0\leqslant z\leqslant L$$
$$t_1=t_3, \quad q_1=q_3 \quad \text{at} \quad r=r_0, \quad z_0\leqslant z\leqslant L \quad (6)$$

$$q_3=\frac{q_{V1}r_0}{2}\left(\frac{r_2^2}{r_0^2}-1\right), \quad m_f>0$$
$$q_3=\lambda_{r3}\frac{\partial t_3}{\partial r}+r_v J_{\mathrm{dep}}f_1, \quad m_f=0 \quad (7)$$

$f_1=\exp[1-(t_w/t_s)^2]$ is the function taking into account the fraction of the deposition flow due to evaporation [45].

3.2. The mass transfer equation for a flow of droplets, without accounting of diffusive transfer along the channel for the ith group is

$$c_i\left(\frac{\partial v_{ri}}{\partial r}+\frac{v_{ri}}{r}+\frac{\partial v_{r1}}{\partial z}\right)+v_{ri}\frac{\partial c_i}{\partial r}+v_{zi}\frac{\partial c_i}{\partial z}$$
$$=\frac{1}{r}\frac{\partial}{\partial r}\left(rD_i\frac{\partial c_i}{\partial r}\right)+J_{\mathrm{en},i}-J_{\mathrm{ev},i} \quad (8)$$

where J_{en} and J_{ev} denote the mass sources and sinks due to liquid entrainment from the film surface and evaporation of droplets in the post-burnout region.

The boundary conditions are

$$c_i = \bar{c}_0 k_i f(r), \quad z = 0, \quad 0 \leqslant r \leqslant r_0 \tag{9}$$

$$\frac{\partial c_i}{\partial r} = 0, \quad r = 0, \quad z_0 \leqslant z \leqslant L \tag{10}$$

$$c_i = 0, \quad r = r_0, \quad z_0 \leqslant z \leqslant L \tag{11}$$

where k_i is the mass fraction of droplets of the ith group and $f(r)$ is the cross-section distribution function of the concentration of droplets.

The mass flow rate of liquid in a film, with evaporation from its surface, entrainment and deposition taken into account, is defined as

$$m_f = m_{f,0} + 2\pi r_0 \int_{z_0}^{z} \sum_{i=1}^{N} \left(-D_{ri} \frac{\partial c_i}{\partial r}\bigg|_{r=r_0-\delta} \right) \mathrm{d}z$$

$$-2\pi r_0 \int_{z_0}^{z} \frac{q_{ev}}{r_v} \mathrm{d}z - 2\pi r_0 \int_{z_0}^{z} J_{en}\, \mathrm{d}z. \tag{12}$$

Here

$$m_{f,0} = \bar{c}_0 \bar{u}_0 \pi r_0^2 \frac{1-\Psi_0}{\Psi_0} \tag{13}$$

Ψ_0 is the liquid fraction in the core at the inlet [42].

3.3. The continuity equation of the vapour flow, with mass sources due to the evaporation of droplets in a superheated vapour taken into acount, reads

$$\frac{\partial u_{3z}}{\partial z} + \frac{u_{3r}}{r} + \frac{\partial u_{3r}}{\partial r} = J_{ev} \tag{14}$$

with the boundary condition

$$u_r = \frac{q_{ev}}{r_v \rho''}\left(1 - \frac{\rho''}{\rho'}\right), \quad r = r_0 - \delta, \quad z_0 \leqslant z \leqslant z_{cr}. \tag{15}$$

Taking into account the above assumptions, the problem of finding the velocity field in the carrying medium and in the liquid film under adiabatic conditions can be presented as

$$\frac{1}{r}\frac{\partial}{\partial r} r \left(v \frac{\partial u}{\partial r} - \overline{u'_r u'_z} \right) = \frac{1}{\rho}\frac{\partial p}{\partial z}, \quad 0 \leqslant r \leqslant r_0 - \delta \tag{16}$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \left(v' \frac{\partial u_2}{\partial r} - \overline{u'_{2r} u'_{2z}} \right) = \frac{1}{\rho'}\frac{\partial p}{\partial z}, \quad r_0 - \delta \leqslant r \leqslant r_0 \tag{17}$$

with the boundary conditions

$$\frac{\partial u}{\partial r} = 0, \quad r = 0$$

$$u = u_2 = u_\delta, \quad \rho v \frac{\partial u}{\partial r} - \rho \overline{u'_r u'_z} = \rho' v' \frac{\partial u_2}{\partial r} - \rho' \overline{u'_{2r} u'_{2z}}$$

$$r = r_0 - \delta. \tag{18}$$

3.4. The convective component in the diffusion equation (8) is determined by the steady-state equation for a moving, spherical, liquid droplet in a vapour flow

$$\pi d_d^3 \rho' (\mathbf{v}\nabla)\mathbf{v} = \frac{\pi d_d^3}{6}(\rho'' - \rho')\mathbf{g}$$

$$-\beta \frac{\pi d_d^2}{8} \rho' |\mathbf{v} - \mathbf{u}|(\mathbf{v} - \mathbf{u}) - \kappa_1 \frac{\pi d_d^3}{6} \rho''$$

$$\times [(\mathbf{v}\nabla)\mathbf{v} - (\mathbf{v}\nabla)\mathbf{u}] + \frac{\pi d_d^3}{6} \rho'' [(\mathbf{v}\nabla)\mathbf{u}]$$

$$+ \frac{\pi d_d^3}{8} \rho'' [\Omega(\mathbf{v} - \mathbf{u})] \tag{19}$$

where κ_1 is the coefficient of the added mass.

3.5. In order to solve numerically the set of equations (1)–(19), these being a simplified version of the annular-dispersed and dispersed flow model, it is necessary to close the equations and present them in dimensionless form. The scales for the unknown and independent quantities are: r_0, L for the lateral and axial coordinate, respectively; $T_{sc} = \bar{q}_{V1} r_0^2 / \lambda_1$ for the temperature, where $\bar{q}_{V1}$ is the mean specific volumetric heat flux in the channel wall; $\bar{u}_0$ for the velocity; ρ' for the mass concentration of the flow of droplets.

Usually, when specifying the energy equation for the wall (1), the heat flux density along the length of the channel is set as

$$q_{F1} = \bar{q}_{F1} f_\eta \tag{20}$$

where f_η is the dimensionless function of the axial heat flux distribution which satisfies the normalizing condition

$$\frac{1}{L}\int_0^L f_\eta\, \mathrm{d}z = 1. \tag{21}$$

For a wall of specified shape (the case of an electrically heated channel with the given f_η), the heat balance equation for a tube element may yield an equation relating the local heat fluxes to the intensity of volumetric sources

$$q_{V1} = \frac{2 q_{F1}}{r_0(\xi_{1ex}^2 - 1)} \tag{22}$$

(here ξ_{1ex} is the dimensionless radius of the external wall of the channel, $\xi_{1ex} = r_{1ex}/r_0$) as well as the relations for the mean values of the above quantities

$$\bar{q}_{F1} = \frac{\bar{q}_{V1} r_0}{2} I, \quad I = \int_0^1 (\xi_{1ex}^2 - 1)\, \mathrm{d}\eta.$$

By introducing the dimensionless quantities $\eta = z/L$, $0 = t/T_{sc}$, $q(\eta) = 2q_F/\bar{q}_{V1} r_0$, with equations (1)–(3) taken into account, the problem of determining the one-dimensional temperature field in a contoured tube wall is presented as

$$\frac{\partial}{\partial \eta}(\xi_{1ex}^2 - 1)\frac{\partial \theta}{\partial \eta} = \Lambda^2 [q(\eta) - f_\eta I] \tag{23}$$

$$\frac{\partial \theta_1}{\partial \eta} = Bi_0 \Lambda (\theta_1 - \theta_{0,sur}) \quad \eta = \eta_0 \tag{24}$$

$$\frac{\partial \theta_1}{\partial \eta} = -Bi_1 \Lambda(\theta_1 - \theta_{1,\mathrm{sur}}) \quad \eta = 1 \tag{25}$$

where $\Lambda = L/r_0$.

In order to determine the heat sink due to droplet evaporation in the region of superheated vapour, equation (4), the following problem will be considered. A single droplet is isolated together with the surrounding spherical volume of superheated vapour of an arbitrary radius r_{cell}. In the superheated vapour in a cell the heat flux per unit time for droplet evaporation is

$$\dot{Q}_{\mathrm{V}i} = \pi d_{\mathrm{d}i}^2 \alpha(t_3 - t_{\mathrm{s}}). \tag{26}$$

The mass lost by evaporation is equal to

$$\dot{m}_i = -\frac{\pi d_{\mathrm{d}i}^2 \alpha(t_3 - t_{\mathrm{s}})}{r_{\mathrm{v}}}. \tag{27}$$

If the concentration and heat sinks are defined respectively as

$$c_i = \left(\frac{d_{\mathrm{d}i}}{d_{\mathrm{cell}\,i}}\right)^3 \rho', \quad q_{\mathrm{V}3i} = 6\frac{\dot{Q}_{\mathrm{V}1}}{d_{\mathrm{cell}\,i}^3} \tag{28}$$

then

$$q_{\mathrm{V}3i} = -\frac{6\alpha(t_3 - t_{\mathrm{s}})}{d_{\mathrm{d}i}\rho'} c_i \tag{29}$$

where α is determined from an experimental formula [35].

After being put in dimensionless form, the problem of determining the temperature field in a vapour-droplet core has the following form

$$\frac{U_{\eta 3}}{\Lambda}\frac{\partial \theta_3}{\partial \eta} = \frac{1}{Pe}\frac{1}{\xi}\frac{\partial}{\partial \xi}\xi \Psi_{\mathrm{T}}\frac{\partial \theta_3}{\partial \xi} U_{\xi 3}\frac{\partial \theta_3}{\partial \xi}$$
$$-\frac{1}{Pe}\frac{6}{d_{\mathrm{bd}}^2}\sum_{i=1}^{N}\frac{C_i Nu_i}{d_{\mathrm{d}i}^2}\theta_3 \tag{30}$$

with the boundary conditions

$$\theta_3 = 0 \quad \text{at} \quad \eta = \eta_0, \quad 0 \leqslant \xi \leqslant 1 \tag{31}$$

$$\frac{\partial \theta_3}{\partial \xi} = 0 \quad \text{at} \quad \xi = 0, \quad \eta_0 \leqslant \eta \leqslant 1$$

$$\theta_1 = \theta_3 = 0, \quad q(\eta) = f_\eta I, \quad \xi = 1,$$
$$\eta_0 \leqslant \eta \leqslant \eta_{\mathrm{cr}}, \quad M_{\mathrm{f}} > 0 \tag{32}$$

$$\theta_1 = \theta_3, \quad q(\eta) = 2\frac{\lambda''}{\lambda_1}\frac{\partial \theta_3}{\partial \xi} - \rho'\frac{u_0 r_{\mathrm{v}}}{\bar{q}_{\mathrm{V}1} r_0}\sum_{i=1}^{N} D_{\xi i}\frac{\partial C_i}{\partial \xi} f_1$$
$$\xi = 1, \quad \eta_{\mathrm{cr}} \leqslant \eta \leqslant 1, \quad M_{\mathrm{f}} = 0. \tag{33}$$

Here

$$d_{\mathrm{bd}} = \frac{d_{\mathrm{bd}}}{r_0}, \quad d_{\mathrm{d}i} = \frac{d_{\mathrm{d}i}}{d_{\mathrm{bd}}}$$

$$\Psi_{\mathrm{T}} = 1 + \frac{a_{\mathrm{tur}}}{a_3}, \quad \frac{a_{\mathrm{tur}}}{a_3} = \frac{Pr}{Pr_{\mathrm{tur}}}\frac{\varepsilon_{\mathrm{v}}''}{v''}$$

where Pr_{tur} is determined according to refs. [36, 46].

It should be noted that the heat sinks in equation (30) and boundary conditions in equation (33) were specified by representing the assembly of droplets by a set of groups with equally sized droplets in each group. The heat sink in equation (30) was considered as a net sink due to evaporation of droplets from different groups. Analogous concepts are used to specify the boundary conditions for equation (33), where a fraction of heat for vaporization of depositing droplets is defined as the total evaporation flux over all the groups.

3.6. Closure and specification of diffusion equation (8) are achieved by representing, in an explicit form, the diffusion coefficient of droplets, heat sinks and sources and by putting equations in dimensionless form. The general expression of the diffusion coefficient is as follows [37]:

$$D \sim \overline{v'^2}\Gamma \tag{34}$$

where $\overline{v'^2}$ is the averaged square fluctuational velocity of a diffusing particle and Γ is the Lagrangian time scale.

If the fluctuation periods of the fluid and of the particle in a turbulent flow are equal, then

$$\frac{D_{\mathrm{p}}}{D_{\mathrm{sur}}} = \frac{\overline{v'^2}}{\overline{u'^2}}. \tag{35}$$

The ratio of the averaged fluctuational velocities defined in ref. [38] as the degree to which the particles can be involved in turbulent fluctuations is essentially the coefficient taking account of the degree of the inertia of particle drifting by diffusion in a turbulent flow. It is clear that for inertia-less inclusions (small particles having the density of the carrying medium), the fluctuational characteristics of the flow and of the inclusions get closer together. Calculations show [39] that a substantial difference in the ratios of velocity fluctuation amplitudes for rotational and translational motion of particles in the flow and, correspondingly, in the diffusion coefficients is observed at low Stokes numbers $N_{\mathrm{S}} = (v''/\omega d_{\mathrm{d}}^2)^{1/2}$ associated with high cyclic frequencies of the carrying medium and large sizes of particles.

In these conditions the diffusion coefficient for an arbitrary group of droplets was taken to be equal to that for inertia-less particles with the correction for their inertia

$$D = D_0\frac{\overline{v'^2}}{\overline{u'^2}} \tag{36}$$

where $D_0 = (0.026/Re^{0.25})\,\bar{u}_0 r_0$ [40], and the values of $E = \overline{v'^2}/\overline{u'^2}$ are calculated according to ref. [39].

The mass sinks due to droplet evaporation in a superheated vapour in equation (8) are determined with the use of relation (29), divided by the vaporization heat, and the mass sources in the flow of droplets are determined by an empirical expression which yields the specific entrainment flow per unit length [41]. After putting the diffusion equation (8) into dimensionless form by introducing the above-given scales and relations for sinks and sources and transforming the linear specific entrainment into the mean volumetric intensity of mass sources, the diffusion equation for an arbitrary group of droplets becomes

$$\frac{U_\eta}{\Lambda}\frac{\partial C_i}{\partial \eta} = \frac{1}{\xi}\frac{\partial}{\partial \xi}\xi D_{i\xi}\frac{\partial C_i}{\partial \xi} - S_{i\xi}U_\xi\frac{\partial C_i}{\partial \xi} - C_i\left[(1-S_{i\xi})\frac{1}{\Lambda}\frac{\partial U_\eta}{\partial \eta} + U_\xi\frac{\partial S_{i\xi}}{\partial \xi}\right] + I_{\mathrm{en},i} + I_{\mathrm{ev},i}. \quad (37)$$

Here

$$I_{\mathrm{ev},i} = \frac{2}{3}Ja\frac{Nu_i}{Pe}\frac{(\rho''/\rho')^2}{[d_{\mathrm{d}i}/(2/r_0)]^2}\frac{c_\mathrm{p}''}{c_\mathrm{p}}C_i\theta_3 \quad (38)$$

where

$$Ja = \frac{c_\mathrm{p}T_\mathrm{sc}}{r_\mathrm{v}}\frac{\rho'}{\rho''}$$

is the Jacob number.

$$I_{\mathrm{en},i} = \frac{J_\mathrm{lin}}{r_0\rho'\bar{u}_0}k_iF(\xi) \quad (39)$$

where J_lin is found according to ref. [41]. It was assumed in calculations that the coefficient k_i, which takes into account the mass fraction of droplets of the ith group in an entrained flow, is equal to the coefficient k_i of the same group in the main flow for which the diffusion equation is considered.

The drop size distribution function in the main and entrained flows is taken to be equal to [30]:

$$\Phi = 4d_\mathrm{d}^2\exp(-2d_\mathrm{d}) \quad (40)$$

$d_\mathrm{d} = d_\mathrm{d}/d_\mathrm{bd}$ where d_bd is determined according to ref. [43].

The mean size of droplets in each group can be found from the relation

$$\bar{d}_{\mathrm{d}i} = \frac{\int_{d_i}^{d_{i+1}} d_\mathrm{d}^3\Phi(d_\mathrm{d})\,\mathrm{d}d_\mathrm{d}}{\int_{d_i}^{d_{i+1}} d_\mathrm{d}^2\Phi(d_\mathrm{d})\,\mathrm{d}d_\mathrm{d}} \quad (41)$$

and the mass fraction of droplets k_i of the ith group is determined by the formula

$$k_i = \frac{\int_{d_i}^{d_{i+1}} d_\mathrm{d}^3\Phi(d_\mathrm{d})\,\mathrm{d}d_\mathrm{d}}{\int_0^\infty d_\mathrm{d}^3\Phi(d_\mathrm{d})\,\mathrm{d}d_\mathrm{d}}. \quad (42)$$

The function of the distribution of mass sources over the channel cross-section $F(\xi)$, equation (39), is taken to be the function which acquires the zero value at the wall, the maximum value at some distance from the wall and a certain, non-zero value in the centre of the channel

$$F(\xi) = k_2(1-\xi)^2\exp\left[-\frac{2}{\Delta_1}(1-\xi)\right] \quad (43)$$

where k_2 is determined from the normalizing condition and $\Delta_1 = m\Delta$ is the optimized coefficient which characterizes the position of the maximum of $F(\xi)$.

The boundary conditions (9) for the diffusion equation incorporate the radial distribution function of the concentration of droplets in the initial cross-section $f(r)$. Most likely, the function should take into account the smooth character of the concentration profile and also follow the assumed conditions for the concentration field of droplets in the centre of the channel and at the wall.

Taking into consideration the velocity contribution to $f(r)$, the following expression was derived in dimensionless form:

$$f(\xi) = (1-\xi^{n_1})\left[2\int_0^1(1-\xi^{n_1})U_\eta\xi\,\mathrm{d}\xi\right]^{-1}. \quad (44)$$

The value of the coefficient n_1 was optimized by comparing the numerical results with the experimental data. The effect of n_1 was found to decrease with the distance from the initial coordinate to the calculated cross-section. It was assumed in calculations that $n_1 = 4$.

After having specified k_i, $f(\xi)$, taking into account that the mean dimensionless concentration of the flow of droplets at the inlet is equal to

$$\bar{C} = \frac{\bar{c}_0}{\rho'} = \frac{\Psi_0}{\rho'}(1-x_0)\Big/\left(\frac{1-x_0}{\rho'}+\frac{x_0}{\rho''}\right) \quad (45)$$

the dimensionless boundary conditions for the diffusion equation may be written as

$$C_i = \bar{C}_0k_if(\xi), \quad \eta = 0, \quad 0 \leqslant \xi \leqslant 1 \quad (46)$$

$$\frac{\partial C_i}{\partial \xi} = 0, \quad \xi = 0, \quad \eta_0 \leqslant \eta \leqslant 1 \quad (47)$$

$$C_i = 0, \quad \xi = 1, \quad \eta_0 \leqslant \eta \leqslant 1. \quad (48)$$

The equation of the liquid balance in a film (12), after being non-dimensionalized, with the re-appearance of the film taken into account (which is possible in the case of non-uniform heat flux distribution along the channel), has the form:

$$M_\mathrm{f} = \frac{1}{2\Lambda}C_0\frac{1-\Psi_0}{\Psi_0} + \int_{\eta_0}^{\eta}\sum_{i=1}^{N}\left(-D_{\xi i}\frac{\partial C_i}{\partial \xi}\bigg|_{\xi=1-\Delta} - I_{\mathrm{en},i}\right)\mathrm{d}\eta - \frac{q_{\mathrm{v}1}r_0}{2\rho'\bar{u}_0r_\mathrm{v}}\times\int_{\eta_0}^{\eta}\left[q(\eta) - 2\frac{\lambda''}{\lambda_1}\frac{\partial\theta_3}{\partial \xi}\bigg|_{\xi=1-\Delta}\right]\mathrm{d}\eta. \quad (49)$$

Here

$$I_\mathrm{en} = \frac{J_\mathrm{lin}}{2\pi r_0\rho'\bar{u}_0}.$$

3.7. With the earlier hypothesis on the evaporation of droplets in a superheated vapour taken into account, the continuity equation of the vapour flow (14) can be put into dimensionless form as

$$\frac{\partial U_\eta}{\partial \eta}+\Lambda\left(\frac{U_\xi}{\xi}+\frac{\partial U_\xi}{\partial \xi}\right)=I_{\mathrm{ev}} \quad (50)$$

with the boundary condition

$$U_\xi = U_{\xi 0} = -\frac{q_{\mathrm{F}}}{\rho'' r_v \bar{u}_0}\left(1-\frac{\rho''}{\rho'}\right), \quad \xi = 1-\Delta \quad (51)$$

where

$$I_{\mathrm{ev}} = \frac{3}{2}\frac{\lambda''}{\rho''}\frac{\bar{q}_{\mathrm{V}} L\theta}{\lambda' r_v d_{\mathrm{bd}} \bar{u}_0}\sum_{i=1}^{N}\frac{Nu_i C_i}{d_{di}^2}.$$

In the above energy and diffusion equations and boundary conditions the value of the most probable droplet diameter in the initial cross-section was calculated according to ref. [43].

When, to express the Reynolds stresses, the eddy viscosities are introduced into the problem of velocity field determination in the core and the film

$$-\overline{u'_r u'_z} = \varepsilon_v \frac{\partial u}{\partial r}, \quad -\overline{u'_{r2} u'_{z2}} = \varepsilon_{v2}\frac{\partial u_2}{\partial r}$$

and also the dimensionless variables

$$U_+ = \frac{u}{v_\tau}, \quad U_{2+} = \frac{u_2}{v_\tau}, \quad \xi = \frac{r}{r_0}$$

$$\Psi_v = \frac{1}{Re_\tau}\left(1+\frac{\varepsilon_v}{v}\right), \quad \Psi_{v2} = \Psi_v \frac{\varepsilon_{v2}+v_2}{\varepsilon_v + v}$$

$$Re_\tau = \frac{v_\tau r_0}{v}, \quad v_\tau = \left(-\frac{r_0}{2\rho}\frac{\partial p}{\partial z}\right)^{1/2} = \left(\frac{\tau_0}{\rho}\right)^{1/2}$$

$$\tau_0 = \rho f \frac{\bar{u}^2}{8} \quad (52)$$

then the problem (16)–(18) takes the form

$$\frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\Psi_v\frac{\mathrm{d}U_+}{\mathrm{d}\xi}\right) = -2, \quad 0 \leqslant \xi \leqslant 1-\Delta \quad (53)$$

$$\frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\Psi_{v2}\frac{\mathrm{d}U_{2+}}{\mathrm{d}\xi}\right) = -2\frac{\rho}{\rho'}, \quad 1-\Delta \leqslant \xi \leqslant 1 \quad (54)$$

$$\frac{\mathrm{d}U_+}{\mathrm{d}\xi} = 0, \quad \xi = 0 \quad (55)$$

$$U_+ = U_{2+} = U_{\Delta+},$$

$$\frac{\rho}{\rho'}\Psi_v\frac{\mathrm{d}U_+}{\mathrm{d}\xi} = \Psi_{v2}\frac{\mathrm{d}U_{2+}}{\mathrm{d}\xi}, \quad \xi = 1-\Delta \quad (56)$$

$$U_{2+} = 0, \quad \varepsilon = 1$$

where

$$\rho = \frac{\rho''[1+x(S_\eta - 1)]}{S_\eta x + (1-x)\rho''/\rho'}. \quad (57)$$

The solutions of equations (53) and (54) with the boundary conditions (55)–(57) are

$$U_+ = U_{\Delta+} + \int_\xi^{1-\Delta}\frac{\xi}{\Psi_v}\mathrm{d}\xi, \quad 0 \leqslant \xi \leqslant 1-\Delta \quad (58)$$

$$U_{2+} = \frac{\rho}{\rho'}\int_\xi^1 \frac{\xi}{\Psi_{v2}}\mathrm{d}\xi, \quad 1-\Delta \leqslant \xi \leqslant 1. \quad (59)$$

After having represented the turbulent exchange coefficients Ψ_v and Ψ_{v2} in explicit form [44] for the relative velocities of the core and the film when calculating the integrals in equations (58) and (59), one finds

$$\frac{U_{\eta 3}}{\bar{U}} = \frac{1}{N}\left\{\frac{1}{m}\frac{\rho}{\rho'}\ln\left|\frac{[P-(1-\Delta)^2]/(P-1)}{[Q-(1-\Delta)^2]/(Q-1)}\right| + \ln\left|\frac{(\xi^2-P)/[P-(1-\Delta)^2]}{(\xi^2-Q)/[Q-(1-\Delta)^2]}\right|\right\}, \quad 0 \leqslant \xi \leqslant 1-\Delta \quad (60)$$

$$\frac{V_{\eta 2}}{\bar{U}} = \frac{1}{N}\frac{1}{m_1}\frac{\rho}{\rho'}\ln\left|\frac{(\xi^2-P)/(P-1)}{(\xi^2-Q)/(Q-1)}\right|, \quad 1-\Delta \leqslant \xi \leqslant 1 \quad (61)$$

where

$$\bar{U} = 2\left[\int_0^{1-\Delta} U_+\xi\,\mathrm{d}\xi + \int_{1-\Delta}^1 U_{2+}\xi\,\mathrm{d}\xi\right]$$

$$N = \frac{1}{m_1}\frac{\rho}{\rho'}\left[P\ln\left|\frac{P-(1-\Delta)^2}{P-1}\right| - Q\ln\left|\frac{Q-(1-\Delta)^2}{Q-1}\right|\right] + Q\ln\left|\frac{Q-(1-\Delta)^2}{Q}\right| - P\ln\left|\frac{P-(1-\Delta)^2}{P}\right|$$

$$P = 0.25 + a_1, \quad Q = 0.25 - a_1$$

$$a_1 = \left(0.5625 + \frac{k(x)[S_\eta x + (1-x)(\rho''/\rho')]}{(\kappa/3)Re_{\tau 3}[S_\eta x + (1-x)\overline{v'^2}/\overline{u'^2}]}\right)^{1/2}$$

$k(x) = [(1-x)v'/v'' + x]/[(1-x)\rho''/\rho' + x]$ is taken from ref. [35], m_1 is the constant of the order of unity, the ratio $\overline{v'^2}/\overline{u'^2}$ is calculated according to ref. [44].

The analysis of equations (60) and (61) shows that they satisfy the limit transitions: when $x \to 0$, $\Delta \to 1$, $\rho \to \rho'$ and expression (60) yields the liquid velocity distribution in a tube. On the other hand when $x \to 1$, $\Delta \to 0$, $\rho \to \rho''$, the first term in expression (60) tends to zero, the second describes the velocity profile of a pure vapour moving in a tube. Finally, when $\varepsilon_v/v \to 0$, formulae (60) and (61) give a parabolic velocity profile for laminar two-phase flow, which, when $x \to 0$, and $x \to 1$, converts into the well-known Poiseuille profile.

In order to make use of the above results to calculate the velocity fields in heated channels, it is necessary to assume that the flow is quasi-stationary, so that the axial velocity profile varies just as the similarity factor equal to the mean relative velocity in the given cross-section. Such variation of the axial velocity is possible only in the case when the profile of the transversal velocity component of the carrying flow develops instantaneously in the given cross-section on evaporation of liquid in the flow.

In these conditions

$$\bar{U}_\eta = \frac{\bar{u}_\eta}{\bar{u}_{\eta_0}} = 1+\kappa_2 \int_{\eta_0}^{\eta} q(\eta)\,\mathrm{d}\eta \Big/ \left(1+\frac{c''_p T_{sc}}{r_v}\theta\right) \quad (62)$$

where

$$\kappa_2 = \frac{q_{V1} r_0 \Lambda}{r_v \rho'' \bar{u}_0}\left(1-\frac{\rho''}{\rho'}\right)$$

is the dimensionless constant.

For the axial velocity in heated channels one gets

$$U_{\eta 3} = \frac{\bar{U}_\eta}{N}\left\{\frac{1}{m}\frac{\rho}{\rho'}\ln\left|\frac{[P-(1-\Delta)^2]/(P-1)}{[Q-(1-\Delta)^2]/(Q-1)}\right| + \ln\left|\frac{(\xi^2-P)/[P-(1-\Delta)^2]}{(\xi^2-Q)/[Q-(1-\Delta)^2]}\right|\right\},$$

$$0 \leqslant \xi \leqslant 1-\Delta \quad (63)$$

$$U_{\eta 2} = \frac{\bar{U}_\eta}{N}\frac{1}{m}\frac{\rho}{\rho'}\ln\left|\frac{(\xi^2-P)/(P-1)}{(\xi^2-Q)/(Q-1)}\right|,$$

$$1-\Delta \leqslant \xi \leqslant 1. \quad (64)$$

The transversal vapour velocity is determined from the continuity equation

$$U_{\xi 3} = \frac{1}{\xi}\left\{U_{0\xi} - \frac{2}{N\Lambda}\frac{\kappa_2 q(\eta)}{1+(c_p T_{sc}/r_v)\bar{\theta}} \times\left[(\xi^2-P)\ln\left|\frac{\xi^2-P}{1-P}\right| - (\xi^2-Q)\ln\left|\frac{\xi^2-Q}{1-Q}\right|\right]\right\}. \quad (65)$$

3.8. The problem of determining the steady-state velocity field of a monodispersed flow of droplets that correspond to the ith group in the assembly, with the use of the above-mentioned scales and account of the forces in equation (19), can be presented in dimensionless form as follows

$$V_\xi \frac{\partial}{\partial \xi}\left[\left(C\frac{\rho'}{\rho''}+\kappa_1\right)V_\eta\right] + \frac{1}{\Lambda}V_\eta\frac{\partial}{\partial\eta}\left[\left(C\frac{\rho'}{\rho''}+\kappa_1\right)V_\eta\right]$$
$$= g\left(1-C\frac{\rho'}{\rho''}\right)\frac{r_0}{\bar{u}_0^2} - \frac{3}{8}\frac{r_0}{r_d}\beta|\mathbf{V}-\mathbf{U}|(V_\eta-U_\eta)$$
$$+(1+\kappa_1)\left(\frac{1}{\Lambda}V_\eta\frac{\partial U_\eta}{\partial\eta}+V_\xi\frac{\partial U_\eta}{\partial\xi}\right)$$
$$-\frac{3}{8}\pi(V_\xi-U_\xi)\left|\frac{\partial U_\eta}{\partial \xi}\right| \quad (66)$$

$$V_\xi \frac{\partial}{\partial \xi}\left[\left(C\frac{\rho'}{\rho''}+\kappa_1\right)V_\xi\right] + \frac{1}{\Lambda}V_\eta\frac{\partial}{\partial\eta}\left[\left(C\frac{\rho'}{\rho''}+\kappa_1\right)V_\xi\right]$$
$$= -\frac{3}{8}\frac{r_0}{r_d}\beta|\mathbf{V}-\mathbf{U}|(V_\xi-U_\xi)$$
$$+(1-\kappa_1)\left(\frac{1}{\Lambda}V_\eta\frac{\partial U_\xi}{\partial\eta}+V_\xi\frac{\partial U_\xi}{\partial\xi}\right)$$
$$+\frac{3}{8}\pi(V_\eta-U_\eta)\left|\frac{\partial U_\eta}{\partial \xi}\right| \quad (67)$$

with the boundary conditions

$$V_\eta = V_\eta(\xi), \quad \eta = 0 \quad (68)$$

$$\frac{\partial V_\eta}{\partial \xi} = 0, \quad V_\xi = 0, \quad \xi = 0. \quad (69)$$

3.9. The set of equations (23)–(69) was solved numerically. The results obtained are discussed in the second part of the paper.

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UN MODELE MATHEMATIQUE BIDIMENSIONNEL DES ECOULEMENTS DISPERSES ANNULAIRES ET DISPERSES—I

Résumé—On présente un modèle mathématiques bidimensionnel de l'écoulement diphasique. La formulation analytique du modèle utilise les équations de conservation de masse, de quantité de mouvement et d'énergie pour la vapeur et les gouttelettes en écoulement, pour le film liquide à la paroi du canal, et aussi un certain nombre de relations supplémentaires pour fermer le système d'équations. Les hypothèses faites sont analysées.

EIN ZWEIDIMENSIONALES MODELL FÜR DEN ÜBERGANGSBEREICH RING-/SPRÜHSTRÖMUNG SOWIE FÜR SPRÜHSTRÖMUNG, TEIL I

Zusammenfassung—Es wird ein zweidimensionales mathematisches Modell der Zweiphasenströmung vorgelegt. Die analytische Modellbildung beinhaltet Massen-, Impuls- und Energiebilanzen für Dampfströmung, Tropfenströmung, Flüssigkeitsfilm und Kanalwand sowie eine Anzahl damit verbundener Beziehungen, um den Satz von Gleichungen zu vervollständigen. Die getroffenen vereinfachenden Annahmen werden analysiert.

ДВУМЕРНАЯ МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДИСПЕРСНО-КОЛЬЦЕВОГО И ДИСПЕРСНОГО ПОТОКОВ — I

Аннотация—Представлена двумерная математическая модель двухфазного потока. Аналитическая формулировка модели включает уравнения сохранения массы, импульса, энергии для потока пара, потока капель, для пленки жидкости и стенки канала, а также ряд вспомогательных соотношений, использованных для замыкания системы уравнений. Рассмотрены и проанализированы вводимые допущения.